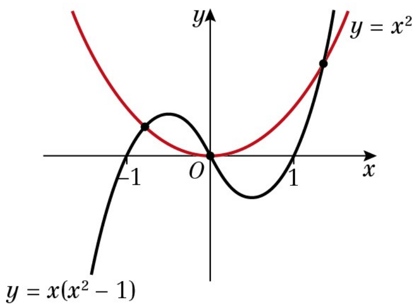
Exercise 4C

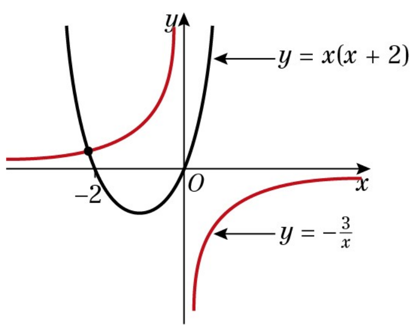
1 a i $y = x^2$ is standard.
$y = x(x^2 - 1)$
$\quad = x(x - 1)(x + 1)$
$0 = x(x - 1)(x + 1)$
So $x = 0$, $x = 1$ or $x = -1$
The curve crosses the x-axis at $(0, 0)$, $(1, 0)$ and $(-1, 0)$.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



ii Three points of intersection

iii Equation: $x^2 = x(x^2 - 1)$

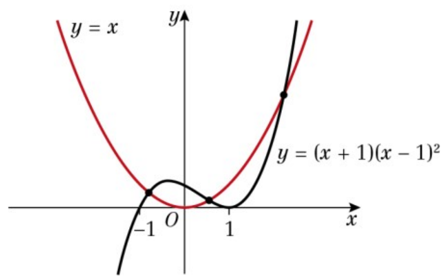
b i $y = x(x + 2)$
As $a = 1$ is positive, the graph has a $\bigvee$ shape and a minimum point.
$0 = x(x + 2)$
So $x = 0$ or $x = -2$
The curve crosses the x-axis at $(0, 0)$ and $(-2, 0)$.
$y = -\dfrac{3}{x}$ is like $y = -\dfrac{1}{x}$ and so exists in the second and fourth quadrants.



ii One point of intersection

iii Equation: $x(x + 2) = -\dfrac{3}{x}$

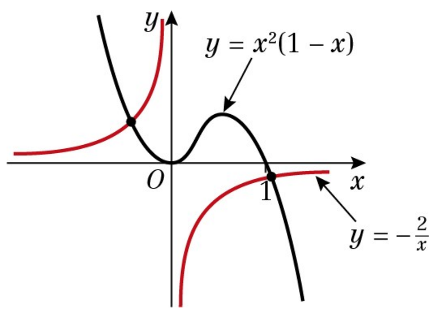
c i $y = x^2$ is standard.
$y = (x + 1)(x - 1)^2$
$0 = (x + 1)(x - 1)^2$
So $x = -1$ or $x = 1$
The curve crosses the x-axis at $(-1, 0)$ and touches it at $(1, 0)$.
$x \to \infty, y \to +\infty$
$x \to -\infty, y \to -\infty$



ii Three points of intersection

iii Equation: $x^2 = (x + 1)(x - 1)^2$

d i $y = x^2(1 - x)$
$0 = x^2(1 - x)$
So $x = 0$ or $x = 1$
The curve crosses the x-axis at $(1, 0)$ and touches it at $(0, 0)$.
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$
$y = -\dfrac{2}{x}$ is like $y = -\dfrac{1}{x}$ and so exists in the second and fourth quadrants.



ii Two points of intersection

iii Equation: $x^2(1 - x) = -\dfrac{2}{x}$

1 e i $y = x(x - 4)$

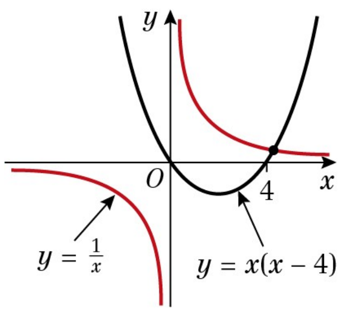
As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

$0 = x(x - 4)$

So $x = 0$ or $x = 4$

The curve crosses the x-axis at $(0, 0)$ and $(4, 0)$.

$y = \frac{1}{x}$ is standard.



ii One point of intersection

iii Equation: $x(x - 4) = \frac{1}{x}$

f i $y = x(x - 4)$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

$0 = x(x - 4)$

So $x = 0$ or $x = 4$

The curve crosses the x-axis at $(0, 0)$ and $(4, 0)$.

$y = -\frac{1}{x}$ is standard and in the second and fourth quadrants.

When $x = 2$,

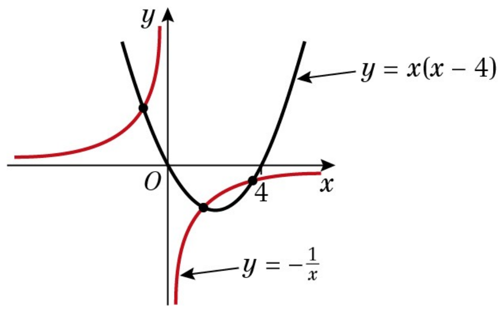
$y = -\frac{1}{x}$ gives $y = -\frac{1}{2}$

$y = x(x - 4)$ gives $y = 2(-2) = -4$

So when $x = 2$, $x(x - 4) < -\frac{1}{x}$

So $y = -\frac{1}{x}$ cuts $y = x(x - 4)$ in the fourth quadrant.

f i



ii Three points of intersection

iii Equation: $x(x - 4) = -\frac{1}{x}$

g i $y = x(x - 4)$

As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.

$0 = x(x - 4)$

So $x = 0$ or $x = 4$

The curve crosses the x-axis at $(0, 0)$ and $(4, 0)$.

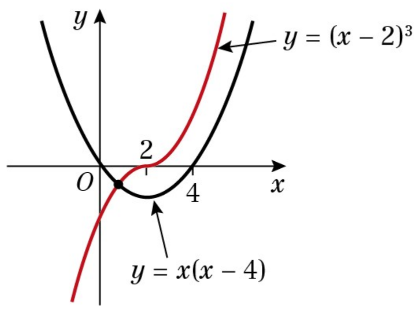
$y = (x - 2)^3$

$0 = (x - 2)^3$

So $x = 2$ and the curve crosses the x-axis at $(2, 0)$ only.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



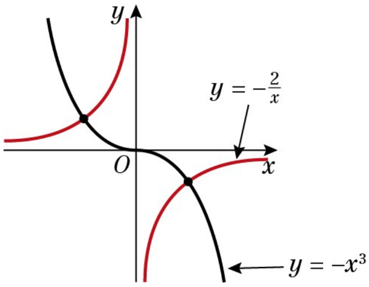
ii One point of intersection

iii $x(x - 4) = (x - 2)^3$

h i $y = -x^3$ is standard.

$y = -\frac{2}{x}$ is like $y = -\frac{1}{x}$ and in the second and fourth quadrants.

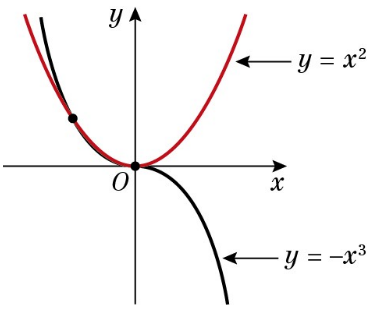
1 h i



ii Two points of intersection

iii $-x^3 = -\dfrac{2}{x}$ or $x^3 = \dfrac{2}{x}$

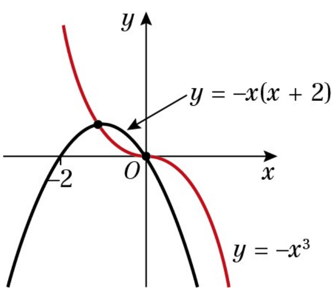
i i $y = -x^3$ is standard.
$y = x^2$ is standard.



ii Two points of intersection.
At (0, 0) the curves actually touch.
They intersect in the second quadrant.

iii $-x^3 = x^2$

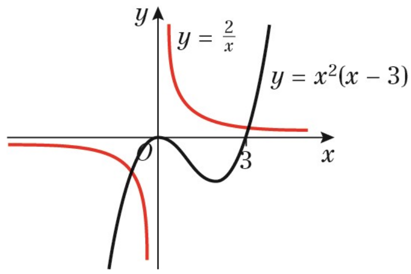
j i $y = -x^3$ is standard.
$y = -x(x + 2)$
As $a = -1$ is negative, the graph has a $\cap$ shape and a maximum point.
$0 = -x(x + 2)$
So $x = 0$ or $x = -2$
The curve crosses the x-axis at (0, 0) and (−2, 0).



ii Three points of intersection

1 j iii $-x^3 = -x(x + 2)$ or $x^3 = x(x + 2)$

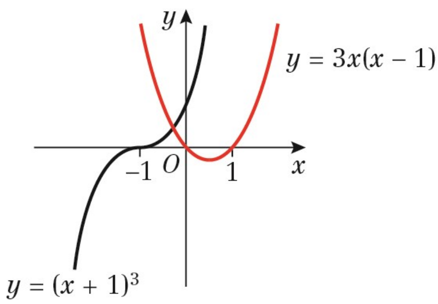
2 a $y = x^2(x - 3)$
$0 = x^2(x - 3)$
So $x = 0$ or $x = 3$
The curve crosses the x-axis at (3, 0) and touches it at (0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = \dfrac{2}{x}$ is like $y = \dfrac{1}{x}$.



b From the sketch, there are only two points of intersection of the curves. This means there are only two values of x where

$x^2(x - 3) = \dfrac{2}{x}$
$x^3(x - 3) = 2$
So this equation has two real solutions.

3 a $y = (x + 1)^3$
$0 = (x + 1)^3$
So $x = -1$ and the curve crosses the x-axis at (−1, 0) only.
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = 3x(x - 1)$
As $a = 3$ is positive, the graph has a $\cup$ shape and a minimum point.
$0 = 3x(x - 1)$
So $x = 0$ or $x = 1$
The curve crosses the x-axis at (0, 0) and (1, 0).



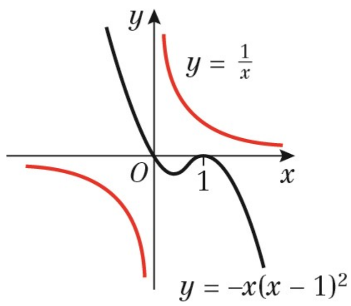
3 b From the sketch, there is only one point of intersection of the curves. This means there is only one value of x where

$$(x+1)^3 = 3x(x-1)$$
$$x^3 + 3x^2 + 3x + 1 = 3x^2 - 1$$
$$x^3 + 6x + 1 = 0$$

So this equation has one real solution.

4 a $y = \dfrac{1}{x}$ is standard.

$y = -x(x-1)^2$
$0 = -x(x-1)^2$
So $x = 0$ or $x = 1$
The curve crosses the x-axis at (0, 0) and touches it at (1, 0).
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to \infty$



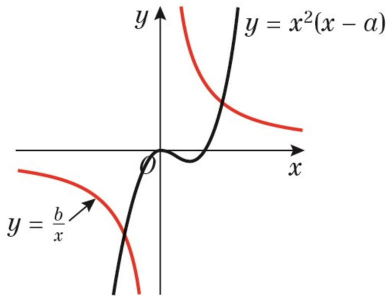
b From the sketch, there are no points of intersection of the curves. This means there are no values of x where

$$\frac{1}{x} = -x(x-1)^2$$
$$1 = -x^2(x-1)^2$$
$$1 + x^2(x-1)^2 = 0$$

So this equation has no real solutions.

5 a $y = x^2(x-a)$
$0 = x^2(x-a)$
So $x = 0$ or $x = a$
The curve crosses the x-axis at $(a, 0)$ and touches it at (0, 0).
$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$
$y = \dfrac{b}{x}$ is a $y = \dfrac{k}{x}$ graph, with $k > 0$.

5 a



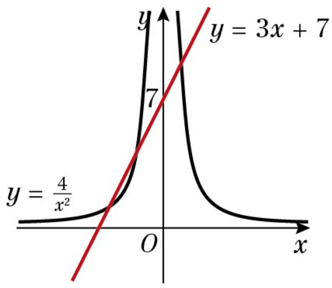
b From the sketch, there are two points of intersection of the curves. This means there are two values of x where

$$x^2(x-a) = \frac{b}{x}$$
$$x^3(x-a) = b$$
$$x^4 - ax^3 - b = 0$$

So this equation has two real solutions.

6 a $y = \dfrac{4}{x^2}$ is a $y = \dfrac{k}{x^2}$ graph, with $k > 0$.

x^2 is always positive and $k > 0$ so the y-values are all positive.
$y = 3x + 7$
$0 = 3x + 7$
So $x = -\dfrac{7}{3}$
$y = 3x + 7$ is a straight line crossing the x-axis at $\left(-\frac{7}{3}, 0\right)$.



b There are three points of intersection, so there are three real solutions to the equation

$$\frac{4}{x^2} = 3x + 7$$

c
$$(x+1)(x+2)(3x-2) = 0$$
$$(x+1)(3x^2 + 4x - 4) = 0$$
$$3x^3 + 7x^2 - 4 = 0$$
$$3x^3 + 7x^2 = 4$$
$$x^2(3x+7) = 4$$
$$3x + 7 = \frac{4}{x^2}$$

6 d $(x+1)(x+2)(3x-2)=0$

So $x=-1$, $x=-2$ or $x=\frac{2}{3}$

Using $y=3x+7$:

when $x=-1$, $y=3(-1)+7=4$

when $x=-2$, $y=3(-2)+7=1$

when $x=\frac{2}{3}$, $y=3\left(\frac{2}{3}\right)+7=9$

The points of intersection are $(-1, 4)$, $(-2, 1)$ and $\left(\frac{2}{3}, 9\right)$.

7 a $y=x^3-3x^2-4x$

$=x(x^2-3x-4)$

$0=x(x-4)(x+1)$

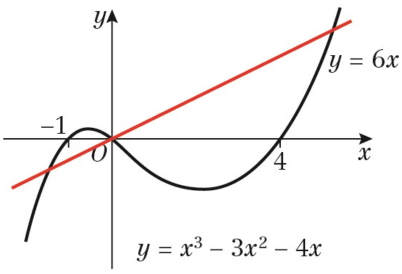
So $x=0$, $x=4$ or $x=-1$

The curve crosses the x-axis at $(0, 0)$, $(4, 0)$ and $(-1, 0)$.

$x\to\infty$, $y\to\infty$

$x\to-\infty$, $y\to-\infty$

$y=6x$ is a straight line through $(0, 0)$.



b $x^3-3x^2-4x=6x$

$x^3-3x^2-10x=0$

$x(x^2-3x-10)=0$

$x(x-5)(x+2)=0$

So $x=0$, $x=5$ or $x=-2$

Using $y=6x$:

when $x=0$, $y=0$

when $x=5$, $y=30$

when $x=-2$, $y=-12$

The points of intersection are $(0, 0)$, $(5, 30)$ and $(-2, -12)$.

8 a $y=(x^2-1)(x-2)$

$=(x-1)(x+1)(x-2)$

$0=(x-1)(x+1)(x-2)$

So $x=1$, $x=-1$ or $x=2$

The curve crosses the x-axis at $(1, 0)$, $(-1, 0)$ and $(2, 0)$.

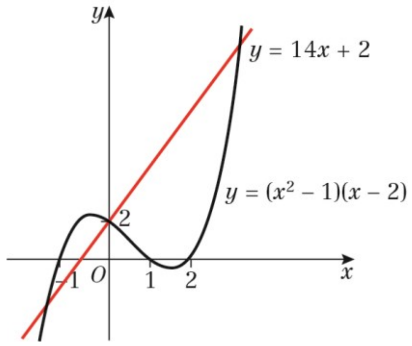
When $x=0$, $y=(-1)^2\times(-2)=2$

$x\to\infty$, $y\to\infty$

$x\to-\infty$, $y\to-\infty$

$y=14x+2$ is a straight line passing through $(0, 2)$ and $(-\frac{1}{7}, 0)$.

8 a



b $(x^2-1)(x-2)=14x+2$

$x^3-2x^2-x+2)=14x+2$

$x^3-2x^2-15x=0$

$x(x^2-2x-15)=0$

$x(x-5)(x+3)=0$

$x=0$, $x=5$ or $x=-3$

Using $y=14x+2$:

when $x=0$, $y=2$

when $x=5$, $y=14(5)+2=72$

when $x=-3$, $y=14(-3)+2=-40$

The points of intersection are $(0, 2)$, $(5, 72)$ and $(-3, -40)$.

9 a $y=(x-2)(x+2)^2$

$0=(x-2)(x+2)^2$

So $x=2$ or $x=-2$

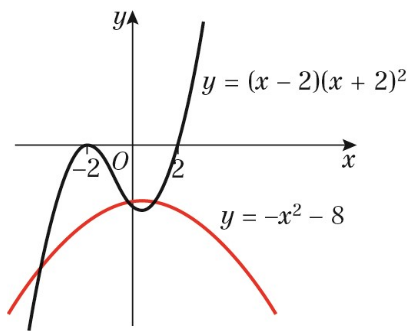
The curve crosses the x-axis at $(2, 0)$ and touches it at $(-2, 0)$.

$x\to\infty$, $y\to\infty$

$x\to-\infty$, $y\to-\infty$

$y=-x^2-8$

As $a=-1$ is negative, the graph has a $\cap$ shape and a maximum point at $(0, -8)$.



b $(x+2)^2(x-2)=-x^2-8$

$(x^2+4x+4)(x-2)=-x^2-8$

$x^3+4x^2+4x-2x^2-8x-8=-x^2-8$

$x^3+3x^2-4x=0$

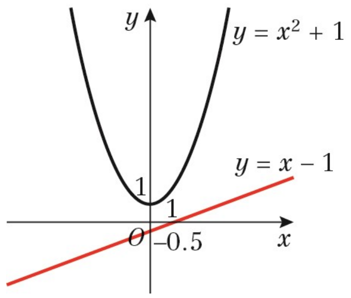
$x(x^2+3x-4)=0$

$x(x-1)(x+4)=0$

So $x=0$, $x=1$ or $x=-4$

9 b Using $y = -x^2 - 8$:
when $x = 0$, $y = -0^2 - 8 = -8$
when $x = 1$, $y = -1^2 - 8 = -9$
when $x = -4$, $y = -(-4)^2 - 8 = -24$
The points of intersection are $(0, -8)$, $(1, -9)$ and $(-4, -24)$.

10 a $y = x^2 + 1$
As $a = 1$ is positive, the graph has a $\bigvee$ shape and a minimum point at $(0, 1)$.
$2y = x - 1$
$y = \frac{1}{2}x - \frac{1}{2}$
This is a straight line passing through $\left(0, -\frac{1}{2}\right)$ and $(1, 0)$.



b The discriminant $b^2 - 4ac = (-1)^2 - 4(2)(3)$
$= -23 < 0$, so there are no real roots

c $x^2 + a = \frac{1}{2}x - \frac{1}{2}$
$2x^2 + 2a = x - 1$
$2x^2 - x + 2a + 1 = 0$
Using the discriminant for two real roots,
$b^2 - 4ac > 0$
$(-1)^2 - 4(2)(2a + 1) > 0$
$1 - 16a - 8 > 0$
$-16a - 7 > 0$
$16a < -7$
$a < -\frac{7}{16}$